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Incorporating Time Varying Volatility in Executive Stock Option Valuation: Implications for US (ASC 718) and Indian (Ind As 102) Accounting

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Abstract: The purpose of this paper is to re-estimate the Black-Scholes option pricing model to include an additional parameter λ. This parameter focuses on time-varying volatility in an attempt to more precisely measure an executive stock option's (ESO) stochastic life feature, a quality which has been inadequately captured by prior research using the constant volatility assumption. By re-estimating the Black-Scholes equation, we find that the value of an ESO may be significantly over or understated in comparison with the conventional Black-Scholes model which does not consider the λ parameter. As a result, it is argued that this modified version of the Black-Scholes model may be of interest to both practitioners and public policy makers. These include the Financial Accounting Standard Boards' (FASB) which governs US GAAP (ASC 718), and the Institute of Chartered Accountants of India (ICAI) which governs Ind AS 102; policy makers that help to shape future accounting policy.

Keywords: ASC 718, Ind AS 102, Executive stock options, λ Parameter, Time-Varying Volatility, Black-Scholes Option model, Compensation Expense, Corporate Governance.

1. INTRODUCTION

The valuation and accounting treatment of Executive Stock Options (ESOs) has long been among the most debated issues in corporate finance and

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accounting policy. The controversy stems from the difficulty of measuring the true economic cost of ESOs to corporations and their shareholders. When the Financial Accounting Standards Board (FASB) first proposed mandatory fair value expensing in 1993, it identified the Black–Scholes option pricing model as a suitable valuation framework. However, the proposal encountered intense resistance, in particularly that from the U.S. technology sector, which argued that the model's reliance on assumptions such as constant volatility and full-term holding would produce unreliable valuations and unfairly depress reported earnings (Fox, 1997). The political backlash led to a temporary retreat and the FASB adopted the disclosure only approach of SFAS 123 in 1995.

The issue resurfaced in the wake of the corporate scandals of the early 2000s and the global movement toward transparency and harmonization. In 2004, the FASB issued *Statement No. 123 (Revised 2004), a share-based payment,* which is now codified as Accounting Standards Codification (ASC) 718. The new standard required public companies to expense ESOs at fair value replacing the permissive treatment of APB Opinion 25. While the binomial-lattice model was theoretically preferred for its ability to incorporate early exercise and vesting restrictions (Carpenter, 1998; Carr and Linetsky, 2001), the Black–Scholes model remained permissible when adjusted for expected term and forfeiture effects (Corey and Schuler, 2011). Post adoption research found that firms responded strategically with many shifting toward less volatile forms of equity compensation, such as restricted stock units (RSUs), to manage reported earnings (Hayes, Lemmon, and Plenk, 2012).

India's trajectory parallels this evolution but also highlights key differences that underscore the global significance of ESO valuation reform. India's accounting standard for share-based payment*, Ind AS 102,* aligns closely with *IFRS 2* (similar to that of ASC 718), mandates fair value measurement at the grant date (Vaidya and Vimalanandan, 2021). However, until the introduction of Ind AS 102 many Indian companies were permitted to use the intrinsic value method. However, this method understated compensation costs and delayed convergence with international practices. The growing use of fair value models under Ind AS 102, especially in India's rapidly expanding technology and IT sectors, has amplified the challenges associated with volatility estimation. This is the same challenge that has persisted in the United States since the adoption of ASC 718.

The **importance** of this issue extends well beyond accounting mechanics. The United States and India are major strategic and financial partners, with significant cross listings via American Depositary Receipts (ADRs) which help program link their capital markets. Inaccurate ESO valuation therefore affects not only firm level financial reporting but also investor decision making, audit assurance, and regulatory comparability across jurisdictions. A valuation model that better captures the economic reality of how volatility evolves over the life of an ESO could help immensely by improving transparency, reducing earnings distortion, and strengthening confidence in reported results.

The fundamental limitation of the Black–Scholes framework is its assumption that volatility (σ) remains constant over the life of the option. This assumption has long been recognized as unrealistic (McKnight and Tomkins, 1999; Heston, 1993; Patton and Sheppard, 2015). Empirical evidence shows that volatility typically declines as firms mature and earnings stabilize (Hasan, Hossain, & Habib, 2017; Liu & Zhang, 2021). Consequently, using a static volatility input tends to overstate ESO value and, in turn, the reported compensation expense for the company.

This paper addresses that problem by introducing a more dynamic volatility input into the Black–Scholes framework. Building on earlier studies that introduced a parameter, λ, to capture the stochastic life of ESOs (Jennergren and Näslund, 1993; Foster et al., 1993), this research redefines λ allowing volatility to decay exponentially over the ESO's life. This modification better reflects the economic behavior of mature more stable firms while maintaining compatibility with the fair value requirements of ASC 718 and Ind AS 102. Our results show that incorporating a decaying volatility parameter produces a substantially lower and more defensible ESO fair value estimate compared to the conventional Black–Scholes model.

The practical implications are significant. For preparers and auditors, this refined model provides a theoretically sound yet operationally simple improvement that enhances the credibility of reported compensation costs. As for policymakers, it offers a means of strengthening international consistency in fair value reporting, particularly between the United States and India given their critical roles in global accounting convergence.

The remainder of this paper proceeds as follows. Section 2 develops the theoretical foundation of the time-varying volatility model and discusses the reinterpretation of λ. Section 3 analyzes the implications of this framework for ESO valuation and financial reporting outcomes. Section 4 concludes with a discussion of the broader policy and practical implications for standard setters and corporate reporting.

2. A MODIFIED VALUATION MODEL

The standard Black-Scholes framework assumes that the volatility estimate (σ) of an underlying stock remains constant throughout the lifespan of an ESO of which has been thoroughly documented (McKnight and Tomkins, 1999). This assumption which simplifies the calculation is generally acknowledged by academics and practitioners as a major theoretical limitation by failing to capture real world market dynamics (Heston, 1993; Patton and Sheppard, 2015). Having said that, our research suggests differently in that a stock's volatility, if anything, seems to decrease over time as a firm matures. This temporal effect, known as decay, is a central characteristic of option valuation which is captured by the option Greek Theta (time decay). Theta measures the rate at which an option's value decreases solely due to the passage of time (Haug, 2007).

Formally, we assume that the instantaneous volatility of the underlying stock follows an exponential decay process over the life of the ESO:

$$\sigma(t) = \sigma_0 \, e^{-\lambda t} \tag{1}$$

where σ_0 is the initial volatility at $t = 0$ and $\lambda > 0$ is the decay parameter governing how rapidly volatility declines over time.

This approach is strongly supported by empirical observation in the post ASC 718 and Ind AS 102 environments. As a firm transition from a high growth, high risk phase to a mature stable entity, its volatility tends to abate. A plausible explanation is that earnings generally become more stable and predictable over time reducing investor skepticism (Ho et al., 1996; Hayes et al., 2012). This economic reality reinforces the need for a parameter (λ) that formally models this time-dependent decay, rendering the simple Black-Scholes model's constant volatility input systematically unreliable for longer dated Executive Stock Options (ESOs).

This temporal reduction in risk which underpins our proposed volatility decay model is further supported by factors beyond market maturity, specifically corporate governance. Empirical observations suggest that as firms mature, their risk profile stabilizes and volatility declines. This effect is not only driven by operational performance but also by the governance structures that guide managerial behavior. Strong internal and external governance mechanisms as documented in large UK public companies have a significant impact on corporate performance and risk-taking behavior (Weir, Laing, & McKnight, 2002). Such oversight can reduce idiosyncratic risk and increase the predictability of earnings providing a strong theoretical and practical rationale for modeling time dependent volatility decay in ESOs. By incorporating a decay parameter (λ) into the Black-Scholes framework, the model captures both the temporal reduction in risk and the influence of governance on firm stability, ultimately enhancing the accuracy of ESO valuation over long maturities.

The classical Black-Scholes option pricing model provides the analytical foundation for this study. However, it should be noted that in practice firms that value ESOs under ASC 718 typically apply the Black-Scholes model using its European style formula but then attempt to incorporate the American style features of employee exercise behavior indirectly by adjusting key inputs. Most notably, they shorten the contractual term to an expected term and may adjust for expected forfeitures and exercise patterns rather than modeling early exercise explicitly. In this study, the λ parameter is introduced to improve the specification of the volatility input and its time decay within this familiar framework. Our approach is fully compatible with the existing practice of using an expected term in which λ operates alongside that adjustment rather than replacing it. Therefore, we do not focus on the formal option style (European vs. American), instead we focus on improving the grant date fair value estimate by refining the volatility input. Under the conventional non-adjusted Black-Scholes model with constant volatility, the grant-date fair value tends to be overstated which in turn inflates the reported compensation expense. By allowing volatility to decay over the life of the ESO through λ, the resulting fair value is lower and more economically realistic. More importantly, this refinement operates entirely within the grant date measurement paradigm of ASC 718 and Ind AS 102. Plain vanilla ESOs are typically measured once at grant and expensed over the vesting period with no subsequent remeasurement or mark-to-market. Our λ-based adjustment therefore improves the initial grant date valuation that underpins the entire expense recognition pattern over the life of the option.

To establish a consistent basis, the standard formulation of the call option value is first presented and is then followed by the introduction of a time dependent volatility decay parameter (λ). The value of a European call option under the Black-Scholes framework is given by:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

or

$$C = (S_0,t) N(d_1) - Ke^{-r(T-t)} N(d_2)$$

where:

$$d_1 = [\ln(S_0 / K) + (r + ½\sigma^2)T] / (\sigma\sqrt{T})$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

variables:

- C = Call option price
- S_0 = Current price of the underlying asset
- K = Exercise (strike) price
- T = Time to maturity (in years)
- t = Current time (valuation date)
- r = Continuously compounded risk-free interest rate
- σ = Volatility of the underlying asset's returns
- $N(d)$ = Cumulative distribution function of the standard normal distribution
- τ = Time remaining to expiration (an alternative notation for T-t)

In the standard model, the volatility (σ) is assumed to be constant. In this study, we replace the static σ with a time-decaying function to create a dynamic specification that better reflects the economic behavior of long-dated ESOs:

$$\sigma(t) = \sigma_0 e^{-\lambda t}$$

The inclusion of the volatility decay parameter (λ) allows the model to capture the empirical decline in firm-specific and idiosyncratic risk over the option's life. This dynamic specification reflects that as time progresses, volatility tends to decrease due to learning effects, managerial influence, and mean reversion in firm performance.

Since the volatility is now time-dependent, the variance term (σ^2) within d_1 and d_2 must be replaced by the average variance (σ^2_{avg}) over the remaining life of the option, from the current time (t) to maturity (T). This is necessary to maintain compatibility with the log-normal distribution assumption for the stock price.

The average variance is calculated as:

$$\sigma^2_{avg} = (1 / (T - t)) \int_t^T \sigma^2(s)\, ds$$

where s denotes calendar time between t and T, and $\tau = T - t$ represents the time remaining to expiration (used interchangeably with T – t in what follows).

Substituting the time-decaying function $\sigma(s) = \sigma_0\, e^{(-\lambda s)}$ into the average variance equation yields the modified volatility input required for the Black-Scholes model. The subsequent section will perform this integration to derive the final analytical closed-form solution for the call option value C(S, t) under time-decaying volatility.

2.1. Delta, Theta, and λ as Dynamic Sensitivities

While the focus of this research is on volatility decay, it is important to recognize that this refinement interacts directly with the fundamental Greeks of option valuation, specifically Delta (Δ) and Theta (Θ). These Greeks which capture the sensitivities of an option's value to changes in the underlying stock price and the passage of time, respectively. The Delta and Theta measure provide an intuitive bridge between the mathematical and behavioral dimensions of executive stock options (ESOs). Theta traditionally represents the rate at which an option's value declines over time assuming that all other factors remain constant. As for the λ parameter, it can be viewed as a second order temporal effect in that it does not simply model the erosion of option value with time but captures the systematic decay of the volatility input that drives that value.

This dynamic interpretation naturally extends to ESOs of any moneyness whether it is at-the-money (ATM), in-the-money (ITM), or out-of-the-money (OTM). Regardless of whether an option has intrinsic value at the grant date a substantial portion of its fair value reflects **time value.** ATM and OTM options are almost entirely composed of time value whereas ITM options include both intrinsic and time value components. Under standard Black-Scholes models time value is calculated assuming constant volatility over the life of the option.

Empirical evidence suggests that firm specific volatility generally declines as the company matures, which accelerates the decay of time value and reduces the probability that OTM or slightly ITM options will finish in the money. By embedding the λ parameter into the Black-Scholes framework we allow volatility to decay steadily over the expected term of the option. This adjustment captures the dynamic erosion of time value across all types of ESOs which, in turn, produces a more realistic and defensible measure of fair value. From an accounting perspective, this ensures that compensation expense recognized under ASC 718 and Ind AS 102 more accurately aligns with the true economic value of the granted options, supporting compliance and providing auditors and practitioners with a robust valuation basis.

Delta, in turn, measures the sensitivity of the option's price to movements in the underlying stock. As volatility declines through the λ parameter, the convexity of the ESO is reduced thus causing Delta to move more rapidly toward unity for ITM options. Consequently, ESOs transition over time from leveraged incentive instruments to more stock like payoff profiles which better reflects the evolving behavioral and economic realities faced by executives. As specified in equation (1), we assume volatility decays exponentially over time before the expiry date of the ESO. Based on the above assumption of volatility and, the hedging equation can be derived by applying Ito's lemma to $C \equiv C(S;\tau;E)$. This will lead to the following parabolic partial differential equation for an American call option $C(S_t;\tau;E)$:

$$\frac{1}{2}\sigma_0^2 \exp\left[-2\lambda(T-t)\right] S^2 \frac{\partial^2 C}{\partial S^2} + (r-\rho) S \frac{\partial C}{\partial S} - rC = \frac{\partial C}{\partial \tau} \tag{2}$$

subject to the expiration condition and the boundary condition:

$$C(S_t;\tau;E) > \max\left[0, S_t - E\right] \text{ for } 0 < S_t < S_t^* \quad 0 \le \tau \le T$$

$$C(S_T;0;E) > \max\left[0, S_T - E\right] \text{ for } 0 < S_T < S_T^*$$

$$C(S_t^*;\tau;E) > \max\left[0, S_t^* - E\right] \text{ for } 0 \le \tau \le T$$

$$\frac{\partial C(S_t^*;\tau;E)}{\partial S_t} = 1 \text{ for } 0 \le \tau \le T$$

where:

- $C(S_t;\tau;E)$ = is the value of an American call option (ESO)
- E = is the exercise (strike) price of the ESO
- $T - t$ = is the time to expiry of the call
- t = is the current time
- S_t^* = is the time dependent optimal exercise boundary (critical stock price) of stock prices λ, at or above it is optimal to exercise the ESO.
- λ = parameter governing the evolution of volatility over the option's life.

3. IMPLICATIONS ON COMPENSATION EXPENSE

Consider the following scenario for Company X, a technology company. As mentioned earlier, we consider λ as a time varying parameter which estimates σ. To highlight the comparative effects under global fair value mandates, we use the long contractual term (T=10) for illustrative purposes. However, please note that current ASC 718 and Ind AS 102 practice would mandate a shorter expected term based on corporate exercise history. Assumptions for the initial fair value estimate are as follows:

- S = 50 (current stock price)
- X = 50 (exercise/strike price)
- r = 0.06 (risk-free rate)
- q = 0.03 (dividend yield of stock X)
- σ_0 = 0.5 (initial stock volatility at t=0)
- T = 10.0 years (option term)
- σ_T = 0.2 (projected terminal stock volatility at t)
- λ = 0.09163 (time-decaying volatility parameter, derived from $\sigma(t) = \sigma_0 e^{-\lambda t}$

In accordance with earlier assumptions, we project that over the ten-year term the firm will mature and its risk profile will stabilize. This would result in a terminal stock volatility of σ_T=0.2. Using the exponential decay model $\sigma(t)=\sigma_0 e^{-\lambda t}$, we can solve for the time-varying decay parameter λ as follows:

$$\sigma_T = \sigma_0 e^{-\lambda T} \Rightarrow 0.2 = 0.5e^{-\lambda 10}$$

$$\lambda = -(\ln 0.2/0.5)/10) = 0.09163$$

This λ value captures the systematic decline in volatility over the life of the executive stock option. This setup implies that the stock's volatility declines over the life of the option starting from an initial σ_0=0.5 to a terminal σ_T=0.2. This reflects the firm's maturation and stabilization of risk. By modeling this decay with λ, the ESO's time value and the executive's early exercise incentives are captured more accurately than under a constant volatility assumption. This ensures that compensation expense better reflects economic reality.

By calculating the ESO based on these estimates, we find that λ has a significant and decreasing effect on the option. This is considerably more than that found under the conventional Black-Scholes model which ignores the λ parameter. Where λ (constant volatility of σ = 0.5 throughout) as shown below has a resulting value of $25.52, the λ = 0.09163 (time-varying volatility) has a value of $20.20.

Even more interesting are the implications λ has on Company X's compensation expense, especially when considering the global accounting context. For example, assume that a typical executive is granted 500,000 options in the year t. As found in Table 1 below, our time-varying λ would reduce the total compensation expense by $2,660,000 or nearly -20.85 percent (25.52 - 20.20/25.52). This difference is material across both ASC 718 (US GAAP) and Ind AS 102 (India) reporting frameworks, where the fair value method is mandatory.

Table 1: Comparative ESO Valuation and Compensation Expense under Constant vs. Time-Varying Volatility (λ)

Valuation Metric	*Conventional B-S Model (Constant Volatility σ = 0.5)*	*Modified B-S Model (Time-Varying Volatility)*	*Difference*	*Percentage Change*
ESO Value (C) per Option	$25.52	$20.20	$5.32	-20.85%
Total Options Granted	500,000	500,000	-	-
Total Compensation Expense	$12,760,000	$10,100,000	$2,660,000	-20.85%
Accounting Regime Relevance	Overstated Fair Value (US/India)	More Accurate Fair Value (US/India)	Impacts US ASC 718 & India Ind AS 102	Material for Regulatory Compliance

It is important to bear in mind that this simple illustration assumes that λ decreases exponentially over time. However, it is entirely possible for λ to increase exponentially as our research regarding volatility tendencies seems to suggest. If this were the case, then compensation expense would increase accordingly. The point here is policy makers in the FASB and ICAI may wish to reappraise their current position on how volatility is estimated under the Black-Scholes model, especially as it remains a common tool used under ASC 718 and Ind AS 102. It is argued that the redefined λ would provide a more precise and defensible estimate of compensation expense.

3.1. Regulatory Context and Policy Relevance Under ASC 718 & Ind AS 102

Although ASC 718 (U.S. GAAP) and Ind AS 102 (India's IFRS-equivalent) both require entities to recognize the grant-date fair value of ESOs as compensation expense, neither framework mandates disclosure or explicit calculation of option Greeks such as Delta, Theta, or Vega. Instead, both standards identify a consistent set of key inputs as sufficient for estimating fair value. These are exercise price, expected term, expected volatility, dividend yield, and the risk-free rate.

Lattice models and Monte Carlo simulations which are endorsed under both ASC 718 and Ind AS 102 incorporate changing Deltas and Thetas recursively by modeling early exercise probabilities and stochastic stock paths. However, their computational complexity has discouraged smaller firms from adopting them. This is particular so in India's technology and mid-cap sectors. The λ modified Black-Scholes model offers a closed form alternative that captures similar realism efficiently. By considering our λ approach which implicitly embeds Delta and Theta dynamics, the results yielded by this approach are consistent with those of lattice-based sensitivity modeling.

From a policy standpoint, incorporating λ refines the fair value hierarchy envisioned by both standards. It aligns with ASC 718's requirement that assumptions reflect the best available information at the grant date and with Ind AS 102's emphasis on reliability and comparability. In practice, using λ could reduce valuation dispersion between firms employing simplified Black-Scholes models and those relying on stochastic simulations. This approach could enhance cross-border consistency in compensation reporting.

In summary, the introduction of λ offers a balance between balances theory and practice. It keeps the Black-Scholes model easy to understand while adding realistic features that reflect how employee stock options behave over time. For accountants and auditors, this provides a fair value method that is both reliable and efficient and, of which, is fully compliant with current standards and more closely aligned with actual economic conditions.

4. CONCLUSIONS

Executive Stock Options (ESOs) remain one of the most complex and misunderstood components of corporate compensation. While they are non-tradable and bound by some restrictions, their economic value is undeniable from both a tool for aligning executive incentives and as a material expense in financial reporting. Yet for decades, accounting standards have struggled to capture this value with any sort of precision. The conventional Black–Scholes model assumes a world that does not exist, among others, that being one of constant volatility.

This paper challenges that static view by introducing a time decaying volatility parameter, λ, into the Black–Scholes framework. The λ model preserves the simplicity that made Black–Scholes the global benchmark. At the same time adding a layer of realism that reflects how firm specific risk and managerial behavior evolve over time. By allowing volatility to decline exponentially, the model produces fair value estimates that are not only theoretically sound but are also more defensible in practice. As demonstrated in our analysis, compensation expense can be materially reduced (by more than 20% in our illustrative case) without sacrificing compliance with **ASC 718** or **Ind AS 102.**

The implications reach far beyond computational refinement. By bridging the gap between **theory and reality**, λ offers accountants, auditors, etc. a powerful yet accessible tool for enhancing transparency and consistency in fair value reporting. For companies, it introduces a disciplined and empirically grounded way to align reported compensation costs with actual economic risk. For regulators, it advances the broader objective of **global accounting convergence,** especially between the United States and India whose financial integration continues to deepen.

In short, λ represents more than a mathematical adjustment, it can also be considered a conceptual shift. It acknowledges that time and behavior matter in that volatility is not static but a living measure of how firms grow, stabilize, and mature. Incorporating this reality into the valuation of ESOs transforms fair value estimation from a theoretical exercise into a truer reflection of corporate economics. If the λ-modified Black–Scholes model were adopted, it could mark the next step in the evolution of equity compensation accounting where transparency and practicality finally converge.

APPENDIX A: BOX NUMERICAL METHOD

Barone-Adesi et al (1997) introduced the Box numerical method to options pricing literature from engineering. Here we apply their Box method to value an ESO. We start by transforming equation (2):

$$\frac{1}{\Psi(S;\tau)}\frac{\partial}{\partial S}\left(\Psi(S;\tau)\frac{\partial C}{\partial S}\right)-\frac{2rCe^{2\lambda\tau}}{\sigma_0^2 S^2}=\frac{2e^{2\lambda\tau}}{\sigma_0^2 S^2}\frac{\partial C}{\partial \tau} \qquad (3)$$

where:

$$\Psi(S;\tau)=S^{\frac{2(r-\rho)\exp(2\lambda\tau)}{\sigma_0^2}}$$

Following Barone-Adesi et al (1997), a grid of size M×N which is constructed for values of $C_n^m = C(n\Delta s, m\Delta t)$ - the value of C at time increment t_m and stock increment S_n, where $t_m = t_0 + m\Delta t$ for m = 0,1, ...M and $S_n = S_0 + n\Delta S$ for $n = 0,1,\ldots,N$.

The values of C_n^m are computed column by column from the left column to the right column and within each column; we solve from bottom to top. Again following Barone-Adesi et-al we set up the following numerical scheme, which is solved using SOR (Successive Over Relaxation) to determine ESO prices.

$$\alpha_n C_n^{m-1} = \chi_n C_{n-1}^m + \eta_n C_n^m + \beta_n C_{n+1}^m$$

$$z_n^m = \frac{1}{\eta_n}\left(\alpha_n C_{n-1}^m + \eta_n C_n^m + \beta_n C_{n+1}^m\right)$$

$$C_n^m = \omega z_n^m + (1-\omega) C_{n-1}^m \quad \text{for} \quad \omega \in (1,2]$$

where:

$$\alpha_n = \frac{2e^{2\lambda m\Delta t}}{\sigma_0^2} X$$

$$\chi_n = -\frac{\Delta t}{\Delta s}\frac{\Psi(S_a)}{\Psi(S_b)}$$

$$\eta_n = \frac{\Delta t}{\Delta s}\left[\frac{\Psi(S_b)}{\Psi(S_n)} + \frac{\Psi(S_a)}{\Psi(S_n)}\right] + \frac{2r\Delta t e^{2m\Delta t}}{\sigma_0^2} X + \frac{2e^{2\lambda m\Delta t}}{\sigma_0^2} X$$

$$\beta_n = -\frac{\Delta t}{\Delta s}\frac{\Psi(S_b)}{\Psi(S_n)}$$

$$X = \frac{1}{S_a} - \frac{1}{S_b}$$

$$S_a = \left(\frac{S_n + S_{n-1}}{2}\right)$$

$$S_b = \left(\frac{S_n + S_{n+1}}{2}\right)$$

APPENDIX B: DERIVATION OF NUMERICAL SCHEME

We use the Euler backward difference to discretize:

$$\frac{\partial C}{\partial \tau} = \frac{C_n^m - C_n^{m-1}}{\Delta t} = \frac{C - C_0}{\Delta t}$$

Further we discretize at each time step:

$$e^{2\lambda\tau} = e^{2\lambda m\Delta t}$$

Thus equation (3) becomes:

$$\frac{1}{\Psi(S;m\Delta t)}\frac{\partial}{\partial S}\left(\Psi(S;m\Delta t)\frac{\partial C}{\partial S}\right) - \frac{2rCe^{2\lambda m\Delta t}}{\sigma_0^2 S^2} = \frac{2e^{2\lambda m\Delta t}}{\sigma_0^2 S^2}\left(\frac{C - C_0}{\Delta t}\right)$$

Re-arranging the above equation and integrating over the range S_b to S_b gives:

$$-\Delta t\int_{S_a}^{S_b}\frac{\partial}{\partial S}\left(\Psi(S)\frac{\partial C}{\partial S}\right)ds+\frac{2r\Delta te^{2\lambda m\Delta t}}{\sigma_0^2}\int_{S_a}^{S_b}\frac{\Psi(S)}{S^2}Cds+\frac{2e^{2\lambda m\Delta t}}{\sigma_0^2}\int_{S_a}^{S_b}\frac{\Psi(S)}{S^2}Cds$$
$$=\frac{2e^{2\lambda m\Delta t}}{\sigma_0^2}\int_{S_a}^{S_b}\frac{\Psi(S)}{S^2}C_0ds$$

We now discretize the integrals as follows:

$$\int_{S_a}^{S_b}\frac{\Psi(S)}{S^2}Cds\approx\Psi(S_n)C_n^m\int_{S_a}^{S_b}\frac{1}{S^2}dS=\Psi(S_n)C_n^m\left(\frac{1}{S_a}-\frac{1}{S_b}\right)=\Psi(S_n)C_n^mX$$

$$\int_{S_a}^{S_b}\frac{\Psi(S)}{S^2}C_0ds\approx\Psi(S_n)C_n^{m-1}X$$

$$-\Delta t\int_{S_a}^{S_b}\frac{\partial}{\partial S}\left(\Psi(S)\frac{\partial C}{\partial S}\right)ds\approx-\frac{\Delta t}{\Delta S}\Psi(S_b)C_{n+1}^m+\frac{\Delta t}{\Delta S}\left(\Psi(S_b)+\Psi(S_a)\right)C_n^m-\frac{\Delta t}{\Delta S}\Psi(S_a)C_{n-1}^m$$

Substituting the above discretizations into our original equation gives:

$$\left\{-\frac{\Delta t}{\Delta s}\frac{\Psi(S_b)}{\Psi(S_n)}\right\}C_{n+1}^m$$
$$+\left\{\frac{\Delta t}{\Delta s}\left[\frac{\Psi(S_b)}{\Psi(S_n)}+\frac{\Psi(S_a)}{\Psi(S_n)}\right]+\frac{2r\Delta te^{2m\Delta t}}{\sigma_0^2}X+\frac{2e^{2\lambda m\Delta t}}{\sigma_0^2}X\right\}C_n^m$$
$$\left\{-\frac{\Delta t}{\Delta s}\frac{\Psi(S_a)}{\Psi(S_n)}\right\}C_{n-1}^m$$
$$=\left\{\frac{2e^{2\lambda m\Delta t}}{\sigma_0^2}X\right\}C_n^{m-1}$$

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